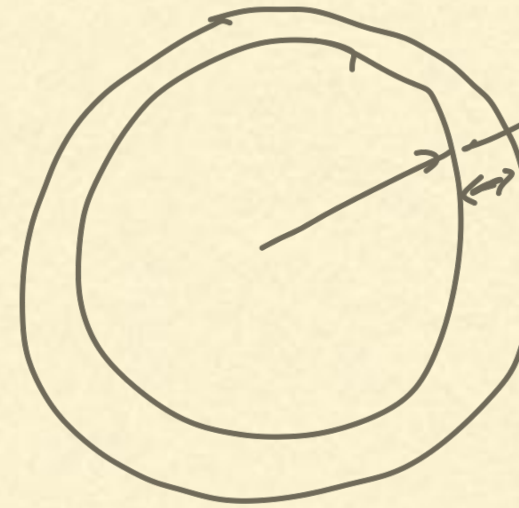
Ex 5.7



$$\tau_\nu = \kappa_\nu \rho \bar{l} = 1 \Rightarrow \bar{l} = \frac{1}{\kappa_\nu \rho}$$

eq 1.23 $P_{rad} = \frac{f}{c}\cos\theta^{\swarrow 0} = \frac{f}{c} = [\text{eq } 1.10] = \frac{L}{4\pi r^2 c}$

↑ Radiation pressure on the outer layer

$$P_{grav} = \frac{F}{4\pi r^2} = \frac{1}{4\pi r^2}\frac{G M_\star}{r^2} m_{shell} =$$

$$= \frac{G M_\star (\rho \cdot V)}{4\pi r^2 r^2} = \frac{G M_\star \rho \overbrace{\cancel{4\pi r^2} l}^{V}}{\cancel{4\pi r^2}\, r^2}$$

$$= \frac{G M_\star \rho \bar{l}}{r^2} = \frac{G M_\star \cancel{\rho}}{r^2}\frac{1}{\kappa_\nu \cancel{\rho}} = \frac{G M_\star}{r^2 \kappa_\nu}$$

Balance P_{grav} & P_{rad}

$P_{rad} = P_{grav}$: $\frac{L}{4\pi r^2 c} = \frac{G M_*}{r^2 \kappa_\nu}$

$\Rightarrow L = \frac{4\pi c G M_*}{\kappa_\nu} = L_{eddington}$

if $L > L_{edd}$ $\Rightarrow$ star will lose mass through strong wind due to P_{rad}

For high T, L, M dominant source of opacity is e^- scattering : $\kappa_\nu = \kappa_{es} = 0{,}2\,(1+X)$

$X = 0{,}707$

$\Rightarrow$ in stellar units $L_{edd} = 3{,}8 \cdot 10^4 \frac{M_*}{M_\odot} \cdot L_\odot$ eq 5.22

cf

$M_* \simeq 136\, M_\odot$

$L_{*obs} = 4{,}5 \cdot 10^6 L_\odot$

$L_{edd*} = ?$

$5{,}2 \cdot 10^6 L_\odot$

good agreement

Eddington limit can be used on e.g AGN

$$M_{BH} \sim 10^8 - 10^{10} M_\odot$$

material

orbiting / accreting

T very hot

v high given energy

If $P_{rad} > P_{grav}$, material blow away

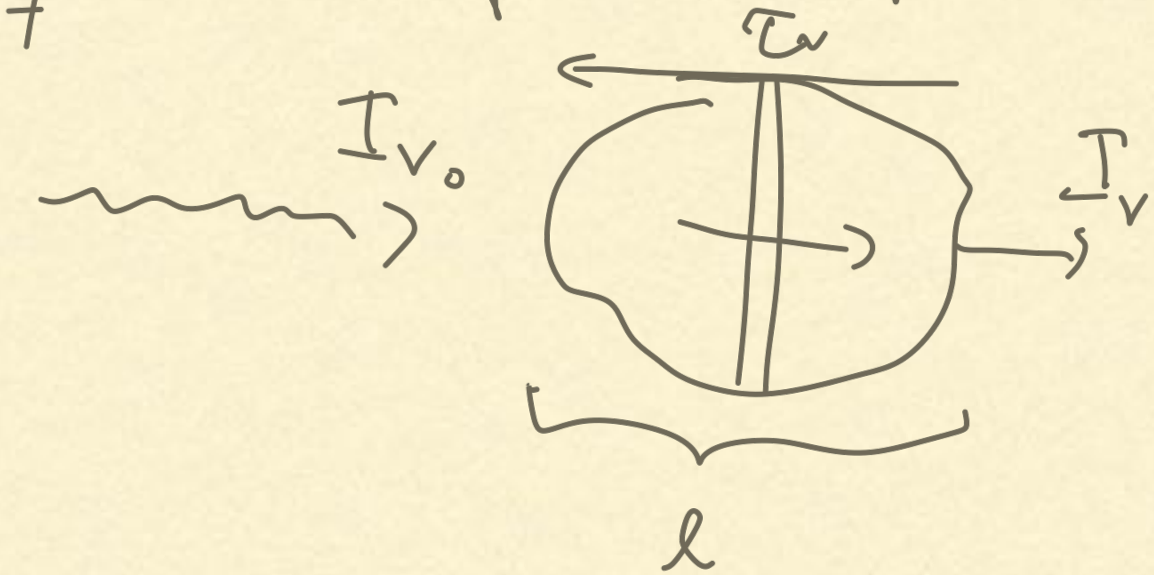
Most BH believed to radiate close to Eddington limit.

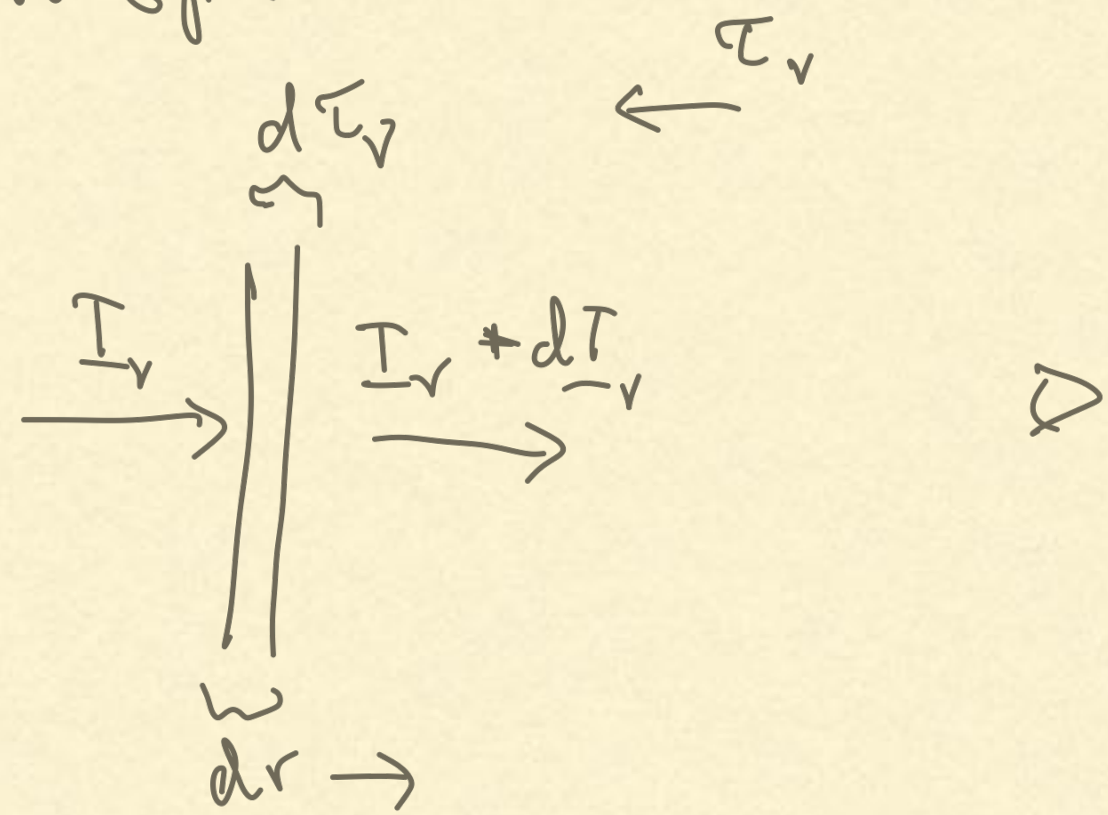
eq 5.22 $L_{edd} = 3.8 \cdot 10^4 \frac{M_*}{M_\odot} L_\odot$

L_{AGN} typical $\sim 10^{45} - 10^{47}$ erg/s $\Rightarrow M_{BH} \approx$ $W^7 - 10^9 M_\odot$

6.1 - read — Radiative transfer most imp in astroph.

Equation of transfer





$$dI_\nu = dI_{\nu\,loss} + dI_{\nu\,gain} \qquad \text{eq. 6.1}$$

+ or −

all sc. & abs processes "absorptions"

all processes inside adding photons emission from matter

@ the ν being observed

$$dI_\nu = -\alpha_\nu I_\nu \, dr + j_\nu \, dr$$

abs coeff $[cm^{-1}]$ (5.8) — emission coefficient $[erg/s \; cm^3 \; Hz \; sr]$

$(\alpha_\nu = \kappa_\nu \rho = n \sigma_\nu)$

Use eq 5.8 $d\tau_\nu = -\alpha_\nu \, dr$ rearrange

$\Rightarrow$ 2 forms

① $\dfrac{dI_\nu}{dr} = -\alpha_\nu I_\nu + j_\nu$ (just divide by dr)

② $\dfrac{dI_\nu}{d\tau_\nu} = I_\nu - \dfrac{j_\nu}{\alpha_\nu} = I_\nu - S_\nu$ (just divide by α_ν too)

$\uparrow = \dfrac{j_\nu}{\alpha_\nu}$ source function

$S_\nu = \dfrac{j_\nu}{\alpha_\nu}$ $\left[erg/s \; cm^2 \; Hz \; sr \right]$ specific int unit

Solutions to eq of radiative transfer

links I_ν to properties of the matter

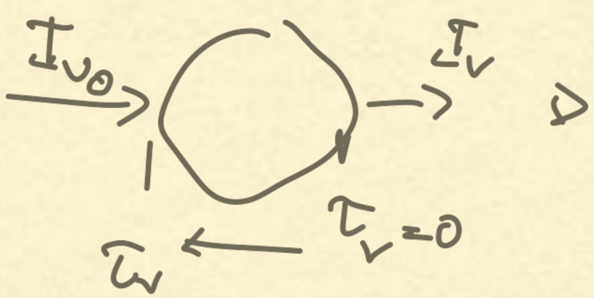
like "heart and soul of astrophysics"

also (key) to deciphering the emission

Ⓐ no cloud $\Rightarrow \alpha_\nu = j_\nu = 0$

① $$\frac{dI_\nu}{ds} = -\cancel{\alpha_\nu} I_\nu + \cancel{j_\nu} = 0 \qquad I_\nu \text{ constant}$$

Ⓑ absorption only



② $$\frac{dI_\nu}{d\tau_\nu} = I_\nu - \frac{\cancel{j_\nu}^{\,0}}{\alpha_\nu} = I_\nu$$

$$\int_{I_\nu}^{I_{\nu_0}} \frac{dI_\nu}{I_\nu} = \int_0^{\tau_\nu} d\tau_\nu \Rightarrow I_\nu = I_{\nu_0} e^{-\tau_\nu}$$

(C) only emissions

$\Rightarrow \alpha_\nu = 0$

(1) $\frac{dI_\nu}{dr} = -\cancel{\alpha_\nu} I_\nu + j_\nu \Rightarrow I_\nu = I_{\nu_0} + \int_0^l j_\nu \, dr$

need functional form of j_ν

if j_ν = same throughout cloud (usually adequate)

$\Rightarrow I_\nu = I_{\nu_0} + j_\nu l$

(D) cloud in true TE

radiation temp is constant $= T_{kin}$

I_ν given by $B_\nu(T)$ Planck fcn — no intensity gradient

$\Rightarrow \frac{dI_\nu}{dr} = 0 \Rightarrow I_\nu$ constant

(1) $0 = -\alpha_\nu B_\nu(T) + j_\nu \Rightarrow B_\nu(T) = \frac{j_\nu}{\alpha_\nu} \equiv S_\nu = I_\nu$ ← source fcn

Kirchhoff's law of radiation

eq 6.12