Recap cross-sections

Now, cross-sections σ (table 5.1)

3 cases

1) free e^- scattering

2) bound e^- — " — ← more complex

3) dust — " —

---

1) a) Thomson $\sigma = \sigma_T = 6.65 \cdot 10^{-25}\ cm^2$

b) Compton (photon energy higher than rest mass energy of e^-; often γ-ray photons)

$\sigma = \sigma_{KN}$ = "Klein-Nishima" cross section

quantum & relativistic effects

decrease in σ with increasing photon energy.

$\Rightarrow$ less efficient scattering when $\nu \uparrow$

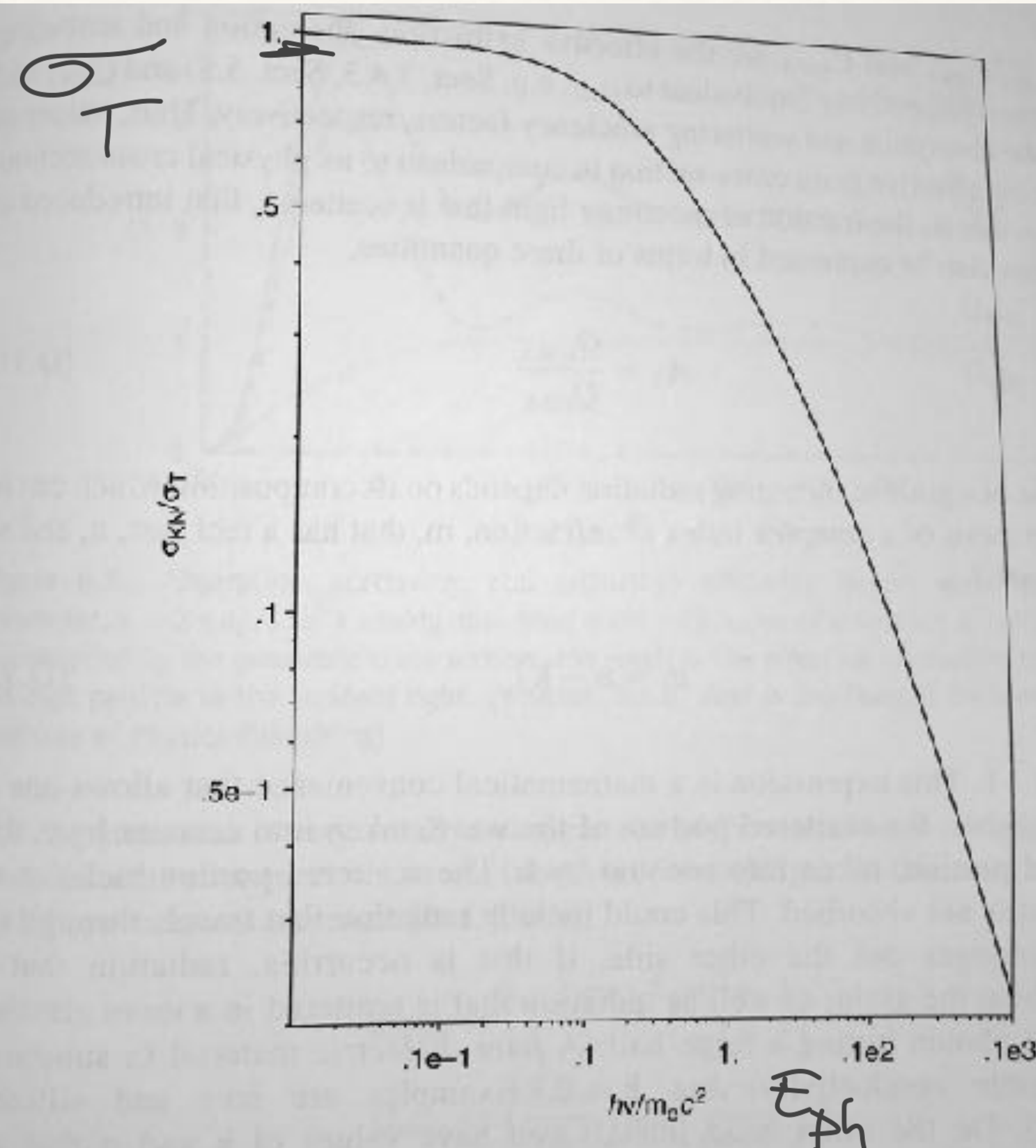


Figure D.4. Ratio of the Klein–Nishina cross-section to the Thomson cross-section, σ_{KN}/σ_T as a function of the parameter, $h\nu/m_e c^2$, showing the decrease in cross-section with increasing photon energy.

2) Scattering from bound e^-

a) oscillator model

$\nu \gg \nu_0$ ← natural ν of oscillator

(ν of radiation field)

e^- responds as if free

$\sigma_s = \sigma_T$

b) $\nu \sim \nu_0$ (classical) — resonance scattering

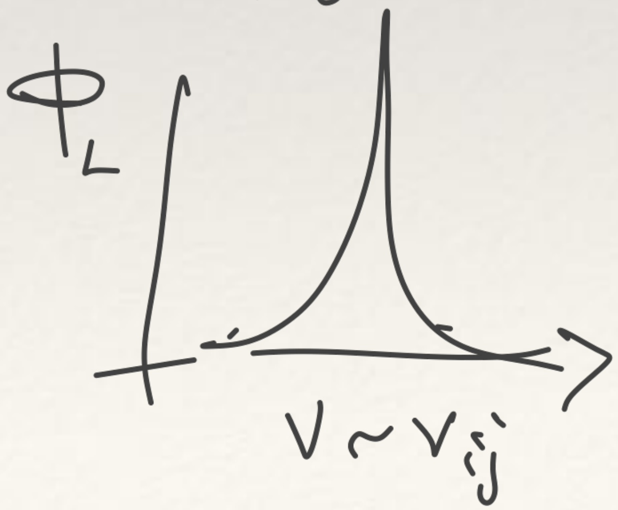
$\nu \sim \nu_{ij}$ (quantum)

also gives a polarized signal

$\sigma_s \propto \Phi_L(\nu)$ Lorentz profile

fig D.2

Some probability photon emitted at different ν



c) $\nu \ll \nu_0$ $\quad \lambda \gg \lambda_0$

recall deBroglie model : $n\lambda_0 = 2\pi r$

$\lambda \gg \lambda_0$ same as $\gg r$ (particle size)

Rayleigh scattering (see eq D.10 and discussion)

$\sigma_s \propto \frac{1}{\lambda^4}$

N_2 O_2 size ~0.3nm

$\lambda_{yellow} = 550$nm $\quad \lambda \gg r$

blue scatters more

night sky also blue (fig 5.8)

3) dust appendix D.3

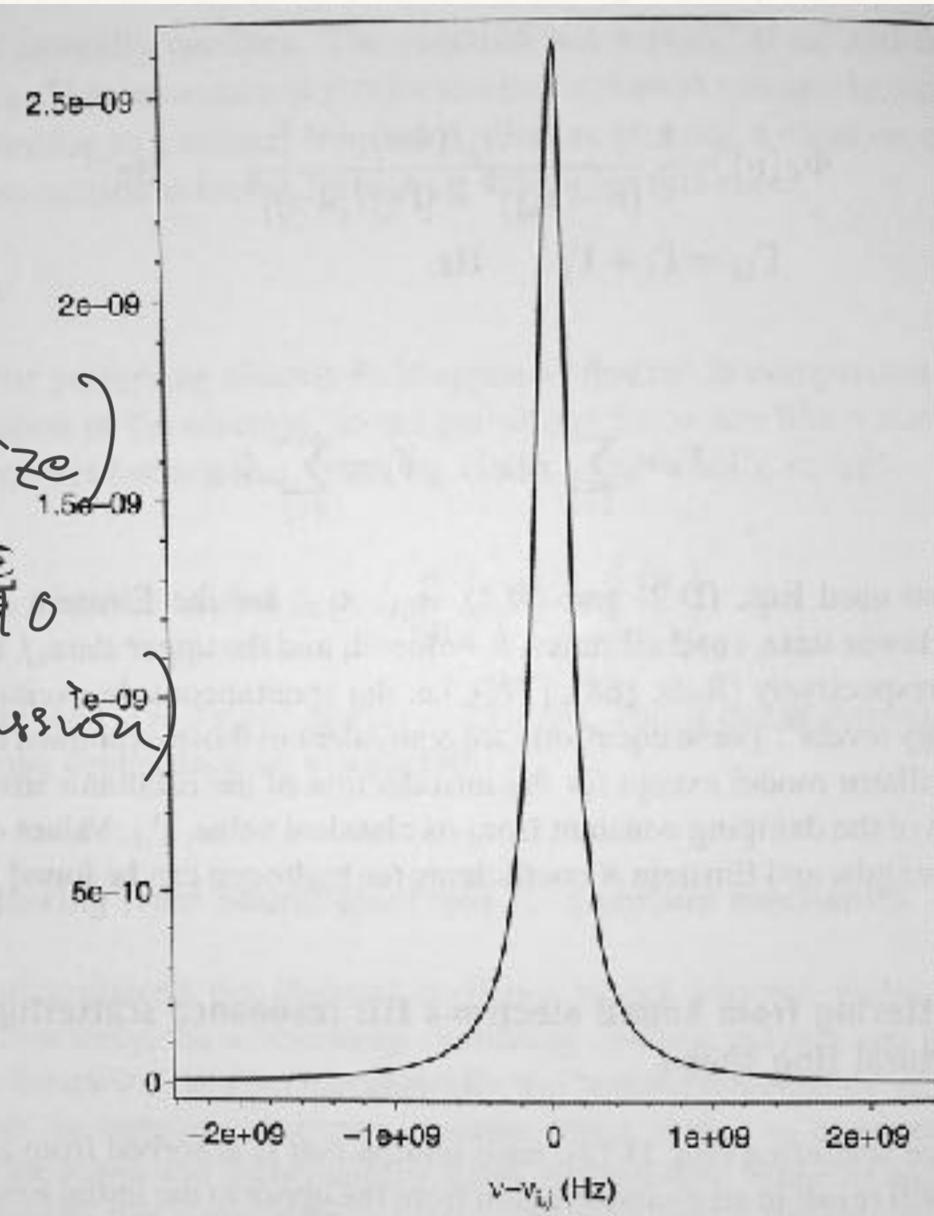


Figure D.2. The Lorentz profile, $\Phi_L(\nu)$, plotted as a function of $(\nu - \nu_{ij})$ for the Ly α line, using the classical damping constant of Eq. (D.5).

Ex 5.2) Find $\bar{l}$ for Lyα photon in HII region

$n = 100/\text{cm}^3$

$T_{HII} = 10^4$ K (Table 3.1)

$\sigma = \sigma_{bb} = 5 \cdot 10^{-14}\ \text{cm}^2$

– resonance bound-bound scattering

$\bar{l} = \frac{1}{n\sigma_{bb}} = 2 \cdot 10^{11}$ cm $< 3 \times R_\odot$

cf to size of HII typically hundreds of pc

Table 5.1. Sample photon interaction cross-sections[a]

Type	Description	Wavelength or energy	Cross-section (cm^2)
σ_T[b]	Thomson scattering	≪ 0.51 MeV	6.65×10^{-25}
σ_{K-N}[c]	Compton scattering	0.51 MeV	2.86×10^{-25}
		5.1 MeV	8.16×10^{-26}
σ_R[d]	Rayleigh scattering (N_2)	532 nm	5.10×10^{-27}
	(CO)	532 nm	6.19×10^{-27}
	(CO_2)	532 nm	12.4×10^{-27}
	(CH_4)	532 nm	12.47×10^{-27}
σ_{b-b}[e]	Ly α (natural)[f]	121.567 nm	7.1×10^{-11}
	Ly α (10^4 K)[g]	121.567 nm	5.0×10^{-14}
$\sigma_{HI \to HII}$[h]	H ionization	13.6 eV	6.3×10^{-18}
σ_{f-f}[i]	free-free absorption	21 cm	2.8×10^{-27}

[a] Cross-sections apply to a single scattering event from a single particle.
[b] Thomson cross-section (Eq. D.2).
[c] Klein–Nishina cross-section for Compton scattering (see Figure D.4).
[d] Rayleigh scattering cross-section for a temperature of 15°C and pressure of 101 325 Pa (Ref. [157]).
[e] Resonance, bound–bound scattering from the line centre.
[f] From the natural line shape using Eq. (D.12) with data from Table C.1. Note that only the permitted transitions have been included (see notes to Table C.1).
[g] As in the previous row but assuming that the line is Doppler broadened (see Sect. 9.3) at the temperature indicated (Ref. [144]).
[h] Photoionization cross-section from the ground state (Eq. C.9).
[i] Free–free absorption cross-section for the conditions: $n_e = 0.1\ \text{cm}^{-3}$ and $T = 10^4$ K. The cross-section will vary with these quantities and also decreases with increasing frequency (Ref. [160]).

Lyα forest

resonance scattering

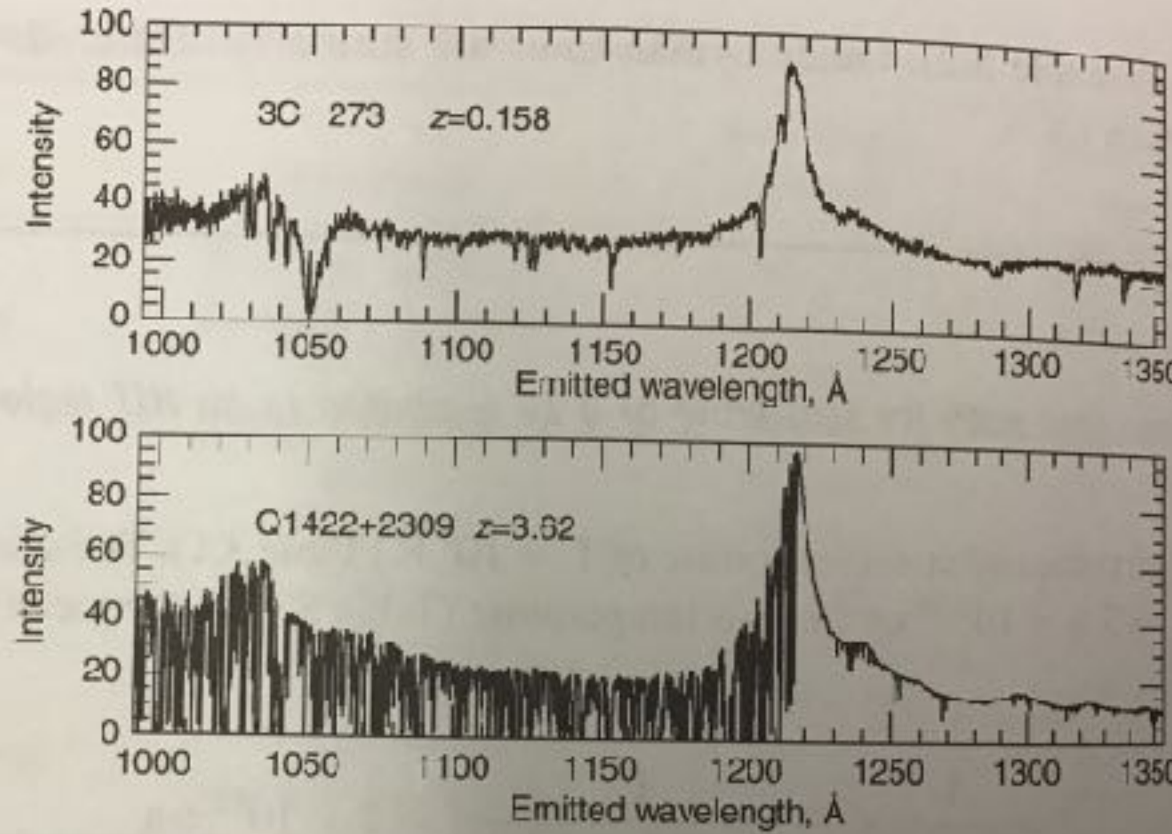
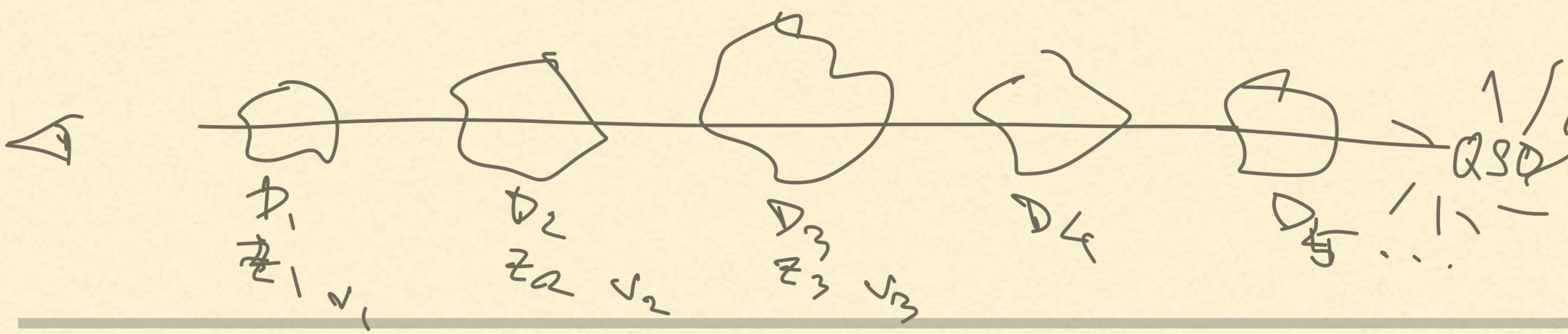


Figure 5.6. Spectra of the quasars, 3C273 (top) and Q1422+2309 (bottom). The quasar, 3C273 is relatively nearby at a redshift of $z = 0.158$ (see Sect. 7.2.2 for a discussion of the expansion redshift) whereas Q1422+2309 is much farther away. The spectra have been aligned so that the Ly α line emitted from the quasars, themselves, is shown at its rest wavelength. In between us and the more distant quasar, Q1422+2309, there are many HI clouds at a variety of distances, creating the many absorption lines seen to the left of the quasar emission. This is called the *Lyman alpha forest*. Most intergalactic gas is, however, ionized. (Reproduced by permission of William Keel)



ABSORPTION

photon loses all its energy

absorption
- kinetic energy gain (incl heating)
  - free-free absorption
  - → e^- ⊕ → e^- photon abs
  - ⇒ e^- different trajectory
  - $\propto \nu, T, n$
- change of state
  - transition
  - photoionization
    - v common
    - (each different element different ionization energy)

PNe, outer edges of galaxies, regions around AGN, HII regions

white dwarf — weak/ionized radiation field — AGN — central hot rate ionization = rate of recombination

ionization equilibrium (cf Temp equilibrium)

$N_i = N_r$

ex 8.4 HII region around a star O8V hot massive

(only those have enough UV to ionize)

any photon of sufficient energy will ionize a neutral atom.

Strömgren sphere R_S

Find R_S - pure hydrogen region

n_e uniform $= 1000/cm^3$

$T = 10^4 K$

$N_i = N_r$

collision rate coefficient table 3.2

recombination when an e^- collides with a ⊕

eq 3.21 total collision rate/unit volume $\nu_{tot} = n_1 n_2 v \sigma = n_1 n_2 \gamma$

$$N_r = \gamma_{tot} \cdot V = n_e n_p \underbrace{v\sigma}_{\gamma} V = n_e n_p \underbrace{\alpha_r}_{\text{total recombination coefficient}} \cdot V = [n_p = n_e]$$

$$= n_e^2 \alpha \frac{4}{3}\pi R_S^3 = N_i$$

rearrange:
$$\underbrace{R_S n_e^{2/3}}_{\text{info about HII region}} = \underbrace{\left(\frac{3}{4\pi}\frac{N_i}{\alpha_r}\right)^{1/3}}_{\text{info abt exciting star}} \qquad \text{eq 5.3}$$

$LHS = \mathcal{U}$

$\mathcal{U} \equiv R_S n_e^{2/3}$ = excitation parameter $\left[\text{pc}/\text{cm}^2\right]$

cgs $\left[\text{cm}/\text{cm}^2\right]$

ionizing luminosity

Now we need N_i can get from $\frac{L_i}{h\nu \; (\nu \geq \nu_i)}$

$$N_i = \frac{L_i}{h\nu \; (\geq \nu_i)} = \begin{bmatrix} 1.10 \; L = 4\pi R_*^2 F \\ 1.14 \; F = \pi I \\ 4.1 \; \text{Planck} \; B_\nu = I \end{bmatrix}$$

$$= 4\pi^2 R_*^2 \underbrace{\int_{\nu_i}^{\infty} \frac{B_\nu(T)}{h\nu} d\nu}_{F/h\nu}$$

OBV

$T_* = 37000 K$ app G G7

$R_* = 6.96 \cdot 10^{11}$ cm

$\nu_i = 3.28 \cdot 10^{15}$ Hz eq C.7

$\Rightarrow N_i = 7.95 \cdot 10^{48}$ photons/s

$n_e = 1000$

α_r (table 3.2)

$= 2.59 \cdot 10^{-13}$ cm^3/s

$\Rightarrow R_s = 1.93 \cdot 10^{18}$ cm

$= 0.63$ pc