

with some departures,  $\star$  are also good BB examples

Measure  $I_\nu$  of object, assume BB  $\Rightarrow$  get  $T$

don't need to know the distance

$I, I_\nu$  are independent of  $D$ , as long as object is resolved

But stars unresolved, get  $f_\nu$

observe  $f_\nu = \underbrace{T \Omega}_{\text{of star}}$



$T_B$  - brightness

temp

= T that when put into

Planck  $\Rightarrow$

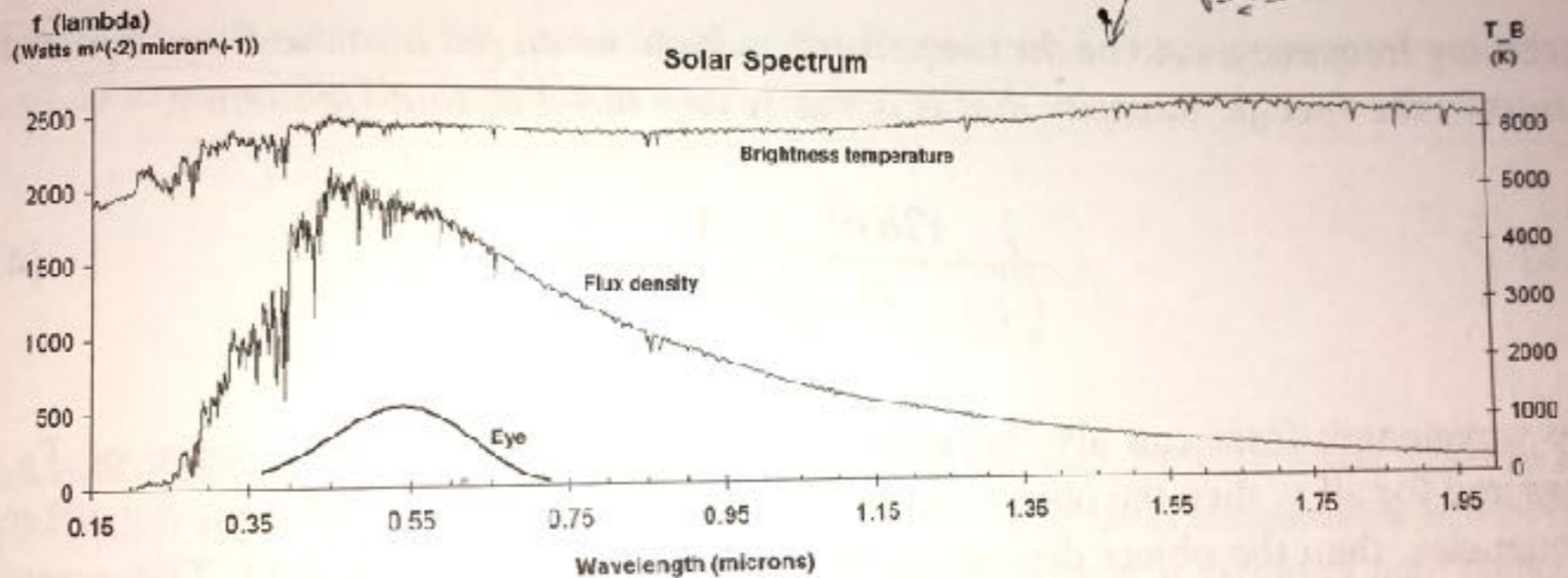
$I_\nu$  that

is measured

at that  $\checkmark$

#### 4.1 BLACK BODY RADIATION

$T_B$  would be a straight horizontal line for a perfect B.B. 127



**Figure 4.5.** The Solar spectrum is again shown (middle curve, left scale) using the same data as in Figure 4.4 except over a more restricted wavelength range and on a linear scale. The brightness temperature (top curve, right scale) is also shown, derived by dividing the flux density by  $\Omega_\odot$  (Eq. 1.13) and then using the Planck formula (the wavelength form of Eq. 4.4) to recover  $T_B$ . The bottom curve, in arbitrary units, is the daylight photon flux response of the human eye smoothed to 30 nm resolution (see also Figure 2.1). (Eye response reproduced by permission of James T. Fulton, 2005, [www.sightresearch.net/luminouseffic.htm](http://www.sightresearch.net/luminouseffic.htm))

$$I_\nu = \frac{2h\nu^3}{c^2} \frac{1}{\left(e^{\frac{h\nu}{kT}} - 1\right)} = B_\nu$$

If seen  $T_B$  for all  $\nu$   
 $\Rightarrow$  true BB

## 4.1.2 Rayleigh - Jeans & Wien's laws

2 approximations to the Planck curve

at low  $\nu$

or rather

$$h\nu \ll kT$$

$$\frac{h\nu}{kT} \ll 1$$

$$R-J: B_\nu(T) \approx \frac{2\nu^2 kT}{c^2}$$

$$\text{ex 4-2 } \frac{h\nu}{kT} < 0,19$$

high  $\nu$

$$h\nu \gg kT$$

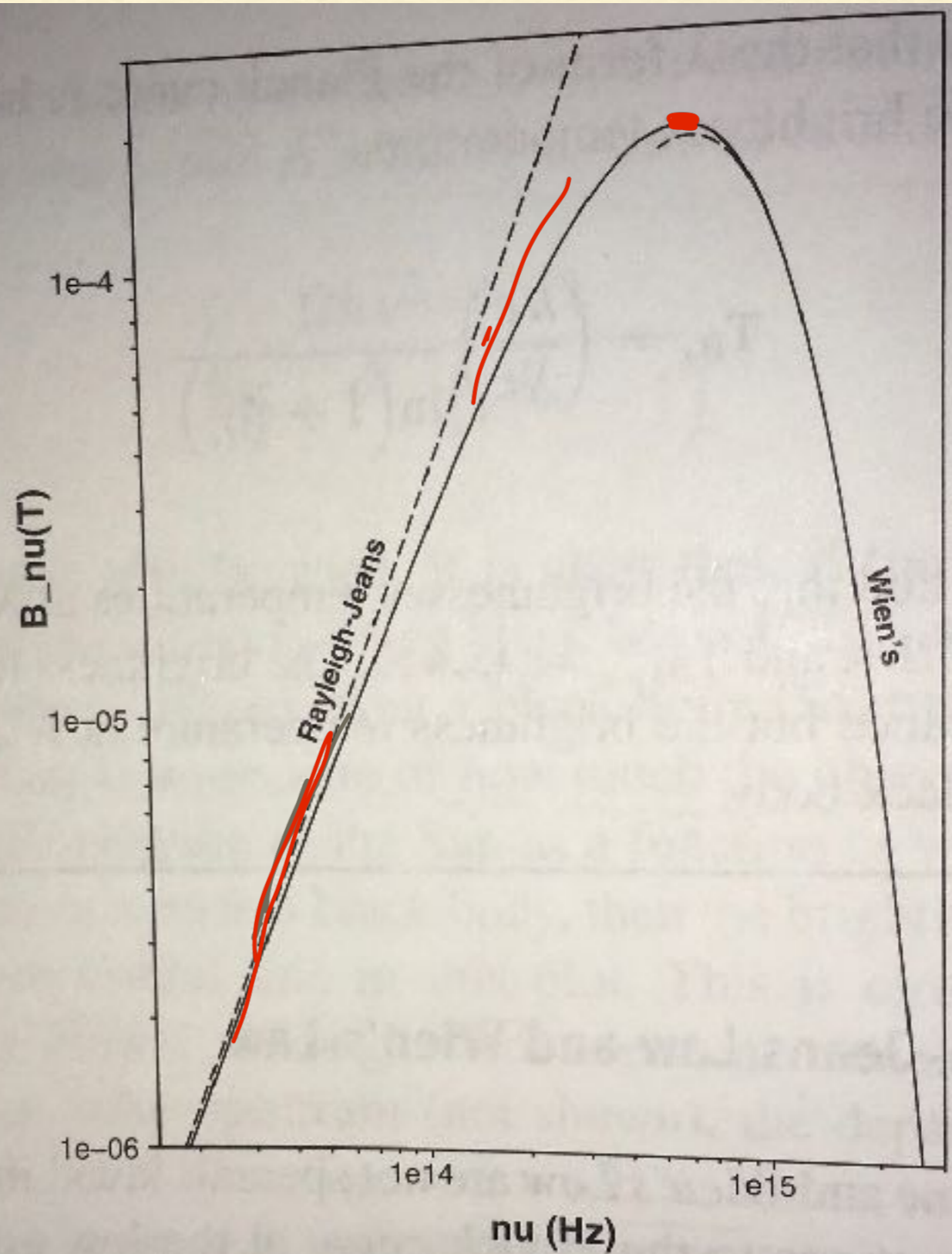
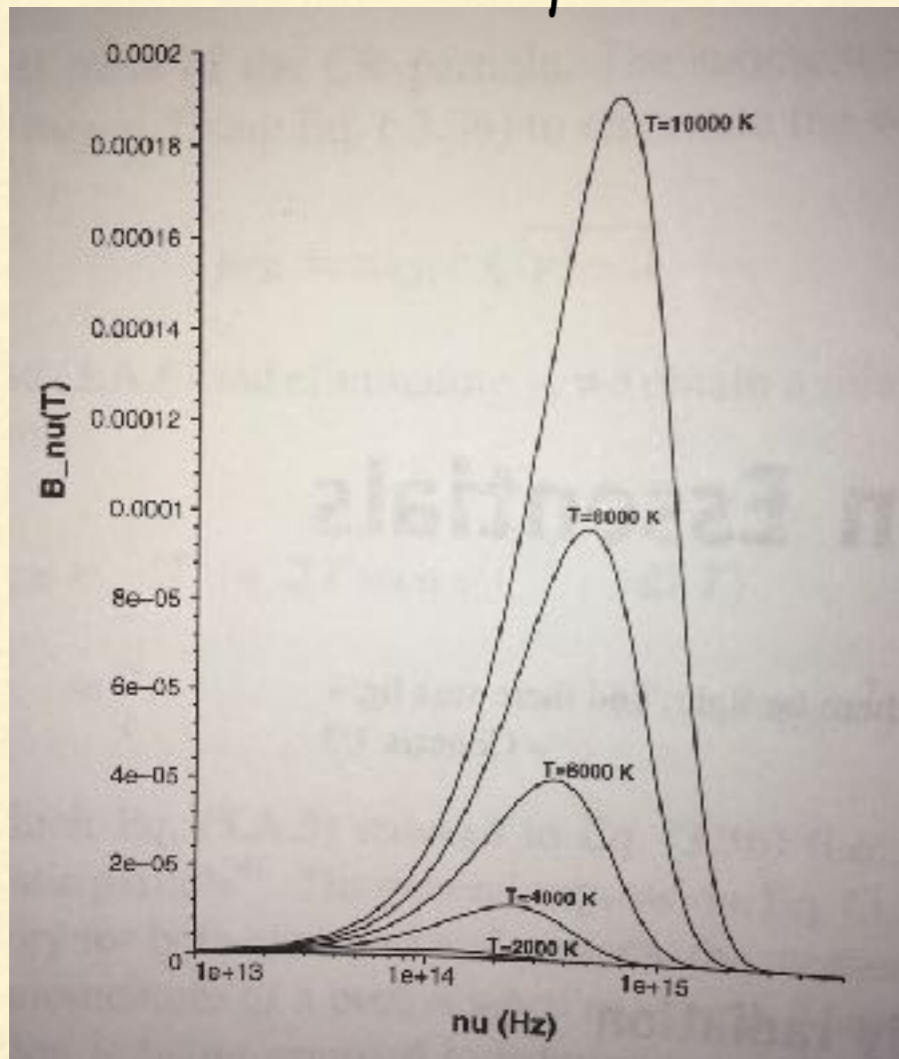
$$\frac{h\nu}{kT} \gg 1$$

$$W: B_\nu(T) \approx \frac{2 h\nu^3}{c^2} \left( e^{-\frac{h\nu}{kT}} \right)$$

$$\frac{h\nu}{kT} > 2,3$$

for 10%  
accuracy or  
better.

Peak of Planck curve -  
 differentiate, set to 0  
 $\Rightarrow \lambda_{max} T = 0.29 \text{ (cgs)}$   
 Wien's displacement  
 as  $T$  higher  $\Rightarrow I_\nu$  - higher  
 and  $T_{peak}$  @ higher  $\nu$



Planck curve, in cgs units, for  $T = 10000 \text{ K}$  (solid curve) ch

Good tool for getting the Temp of unresolved sources.

normally  $B_\nu(T)$  to get  $T$  ← resolved  
unresolved  $\Rightarrow f_\nu$   $f_\nu = B_\nu(T) \Omega$

So - identify  $\nu$  @ peak of spectrum  
and use Wien's displacement law to get  $T$   
(no need for distance)  $\hat{c}$  not always known

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4.1.4 Suppose representative  $T$  over all  $\lambda$  for a  $\star$   
- integrate over  $\nu$  (or  $\lambda$ )  $\left\{ \begin{array}{l} \text{we want} \\ \text{we want} \end{array} \right.$

4.9  $B(T) \int_0^\infty B_\nu(T) d\nu = \mathcal{L} T^4$  [erg/cm<sup>2</sup>s]

To turn this into a flux  $\mathcal{L} = \frac{2 (kT)^4}{15 c^2 h^3}$

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$$\text{eq 1.14 } = F = \pi \cdot I = \pi B(T) = \sigma T_{\text{eff}}^4$$

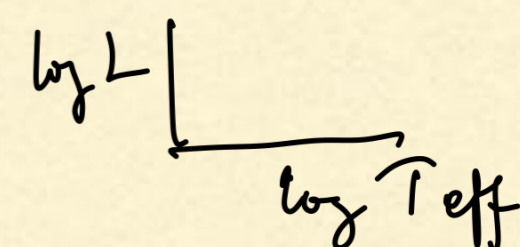
$$\sigma = \text{Stefan-Boltzmann constant} = \pi^2/15 = 5.67 \cdot 10^{-15} \frac{\text{erg}}{\text{cm}^2 \text{K}^4}$$

$$F = \sigma T_{\text{eff}}^4 \quad \left[ \frac{\text{erg}}{\text{cm}^2} \right]$$

$T_{\text{eff}}$  = whatever temp that results in the flux observed from the object.

use this flux to get  $L_{\text{bol}}$  (sphere of radius  $R$ )

$$L = 4\pi R^2 F = 4\pi R^2 \sigma T_{\text{eff}}^4 \quad \text{eq 4.13}$$

$L_{\text{bol}}$  is a strong fun of its  $T_{\text{eff}}$ . 

take log:  $\log L = 4 \log T_{\text{eff}} + 2 \log R + \text{const}$

tells us a lot about placement of stars in  $H-R$

$$\log L \sim M_{\text{abs}}$$

$T \sim \text{colour index } B-V$

$\Rightarrow$  H-R diagram

$\approx$  relationship b/w

lum and temp of a star

if all ~~stars~~ same size

$\Rightarrow$  MS would be a

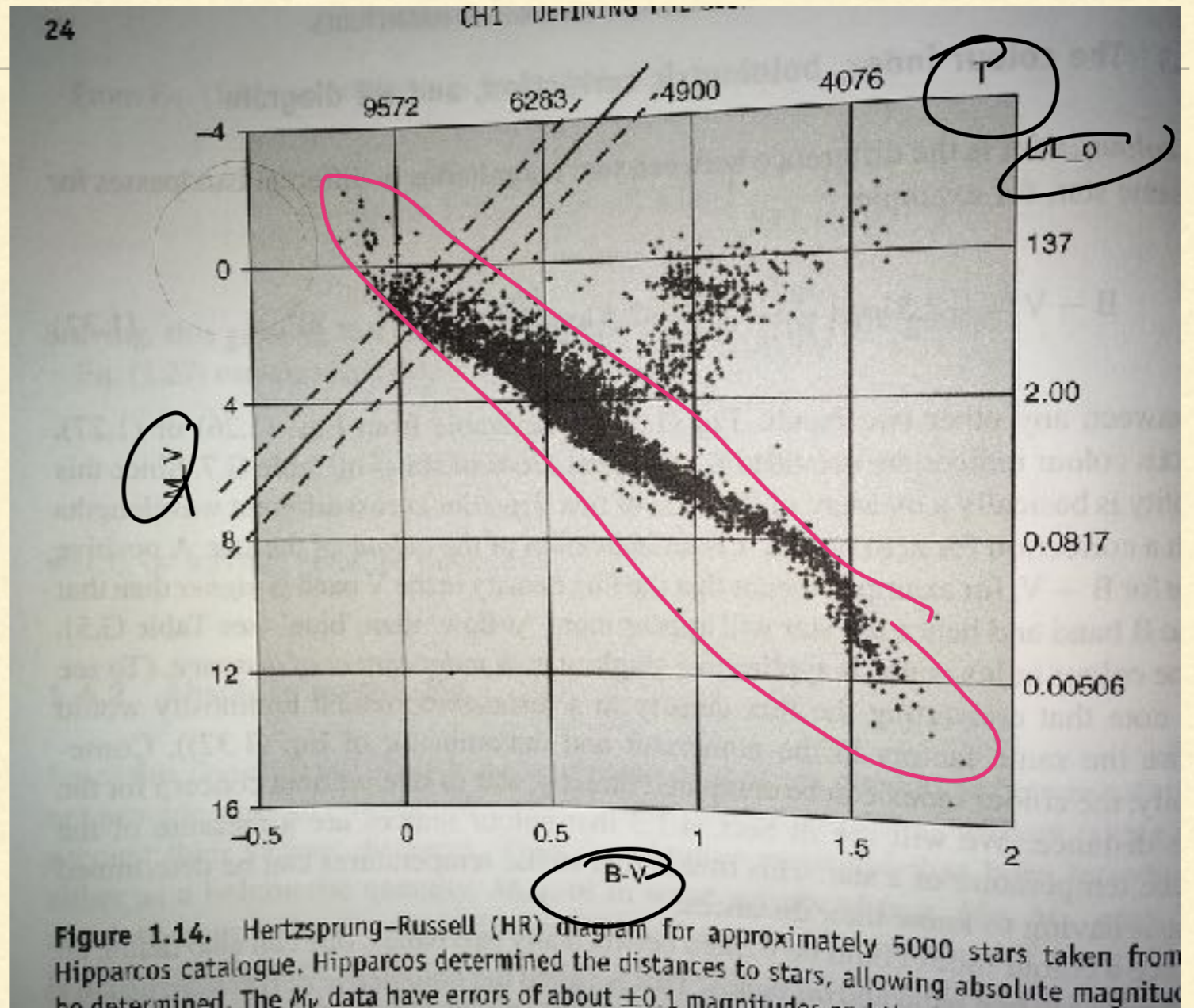
straight line of slope 4

in reality it's steeper

$\Rightarrow$  hotter stars are larger

curvature to MS due to 2<sup>nd</sup> order effects such

as opacity which can affect the radius.



## 4.1.5 Energy density & pressure in stars.

radiation field close to isotropic within star

energy density

$$u = \frac{4\pi}{c} \int I_{\nu} d\nu \quad \leftarrow \text{eq 1.16} \quad I \approx J$$
$$= \frac{4\pi}{c} \bar{I} = \frac{4\pi}{c} B =$$

↑ mean intensity

$$= \frac{4\pi}{c} \sigma T^4 = a T^4$$

For isotropic radiation

↑  $a =$  radiation constant =

$$7.57 \cdot 10^{-16} \text{ erg/cm}^3 \text{K}^4$$

$$P = \frac{1}{3} u \quad (\text{eq 1.22})$$

$$\Rightarrow P_{\text{rad}} = \frac{1}{3} a T^4$$

must be added to  $P_p$  (particle pressure) to determine total  $P$  within star

For low mass stars (☉):  $P_{\text{rad}}$  negligible

For hotter, more massive stars



low mass  $\star$  (0)  $P_{\text{rad}}$  negligible

hotter more massive  $\star$   $P_{\text{rad}}$  depends strongly on  $T$

very high mass  $\star$   $\Rightarrow$  dominant  
 $\Rightarrow$  affects stability of  $\star$

loss of outer layers

ex: Wolf-Rayet stars

(more in 5.4.2)

(see 5.1.2.2)

## 4.2 Grey bodies

Any opaque body which also has some reflectivity - Albedo  $A$

$A=0$  perfectly absorbing  
 $A=1$  perfectly reflecting

Table G.4. Planetary data<sup>a</sup>

Planet	Equatorial diameter (km)	Oblateness	Mass (Earth = 1)	Density (g cm <sup>-3</sup> )	Rotation period <sup>b</sup> (days)	Incl. <sup>c</sup>	Albedo <sup>d</sup> (Bond)	a <sup>e</sup> (AU)	e <sup>f</sup>
Mercury	4 879	0	0.055 274	5.43	58.646	0.0	0.119	0.3871	0.2056
Venus	12 104	0	0.815 005	5.24	243.019	2.6	0.750	0.7233	0.0068
Earth	12 756	1/298	1.000 000	5.52	0.9973	23.4	0.306	1.0000	0.0167
Mars	6 792	1/148	0.107 447	3.94	1.0260	25.2	0.250	1.5237	0.0935
Jupiter	142 980 <sup>g</sup>	1/15.4	317.833	1.33	0.4101 <sup>h</sup>	3.1	0.343	5.2020	0.0490
Saturn	120 540 <sup>g</sup>	1/10.2	95.163	0.69	0.4440	26.7	0.342	9.5752	0.0568
Uranus	51 120 <sup>g</sup>	1/43.6	14.536	1.27	0.7183	82.2	0.300	19.1315	0.0501
Neptune	49 530 <sup>g</sup>	1/58.5	17.149	1.64	0.6712	28.3	0.290	29.9681	0.0086
Pluto <sup>i</sup>	2 390	0?	0.002 2	1.8	6.3872	57.4	0.4–0.6	39.5463	0.2509

<sup>a</sup>Ref. [71] unless otherwise indicated.

<sup>b</sup>Sidereal.

<sup>c</sup>Inclination of equator to orbital plane.

<sup>d</sup>Albedo is the fraction of incident light that is reflected. This table gives the *Bond albedo* (from <http://nssdc.gsfc.nasa.gov>), defined as the total fraction of reflected Sunlight over all wavebands.

<sup>e</sup>Mean distance to the Sun (equal to the semi-major axis of the ellipse).

<sup>f</sup>Mean orbital eccentricity. The perihelion and aphelion distances are given by  $r_{\min} = a(1 - e)$  and  $r_{\max} = a(1 + e)$ , respectively.

<sup>g</sup>At 1 atm (101.325 kPa).

<sup>h</sup>For the most rapidly rotating equatorial region.

<sup>i</sup>At the XXVIth General Assembly of the International Astronomical Union held in Prague in August, 2006, 'Resolution 6' was adopted, indicating that 'Pluto is a "dwarf planet" ... and is recognized as the prototype of a new category of Trans-Neptunian Objects.'

Bond Albedo  $A_b = \int_0^{\infty} A_{\lambda} d\lambda$   
 "grey" because absorption<sup>0</sup>

total fraction of reflected sunlight over all  $\lambda$