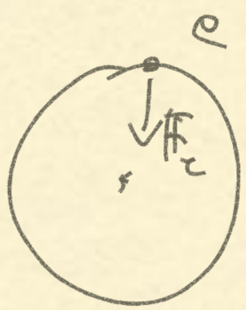
gyrofrequency ν_0 = frequency of gyration of the e^- around the magnetic field line



$$F_e = \frac{e}{c}\left(\mathbf{v}\times\mathbf{B}\right) = \text{Lorentz force}$$

$$F_e = \frac{ev}{c}\underset{\uparrow\,\text{pitchangle}}{B\sin\varphi} = \frac{ev}{c}\underset{\uparrow\,B\sin\varphi}{B_\perp} \qquad \text{eq 8.26}$$

$$= \left[=\text{centripetal force}\right] = \frac{m_e v_\perp^2}{r_0}$$

$\uparrow$ radius of gyration - orbital radius $\perp$ to B

[$v_\perp = v\sin\varphi$ = perpendicular component of v, the one affected by B]

$$\Longrightarrow \frac{eB}{c} = \frac{m_e v_\perp}{r_0}$$

solve for r_0:

$$r_0 = \frac{m_e v_\perp c}{eB} = \frac{v_\perp T}{2\pi}\ ; \quad T = \frac{1}{\nu_0} = \text{period of gyration}$$

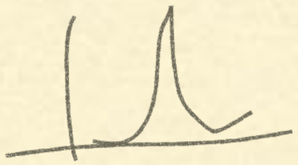
and $v_\perp = \frac{2\pi r_0}{T}$

solve for ν_0 gyro frequency

$$\nu_0 = \frac{eB}{2\pi m_e c} \Rightarrow \underset{\uparrow\,\text{MHz}}{\nu_0} = 2.8\,\underset{\text{Gauss}}{B} \qquad \text{eq } 8.28$$

not dependent on v or E_{kin}, when $v \ll c$

if B const $\Rightarrow$ ν_0 const $\Rightarrow$



in reality B has range of values $\Rightarrow$ "semi-continuous"

only observe ν_0 $\Rightarrow$ get B.

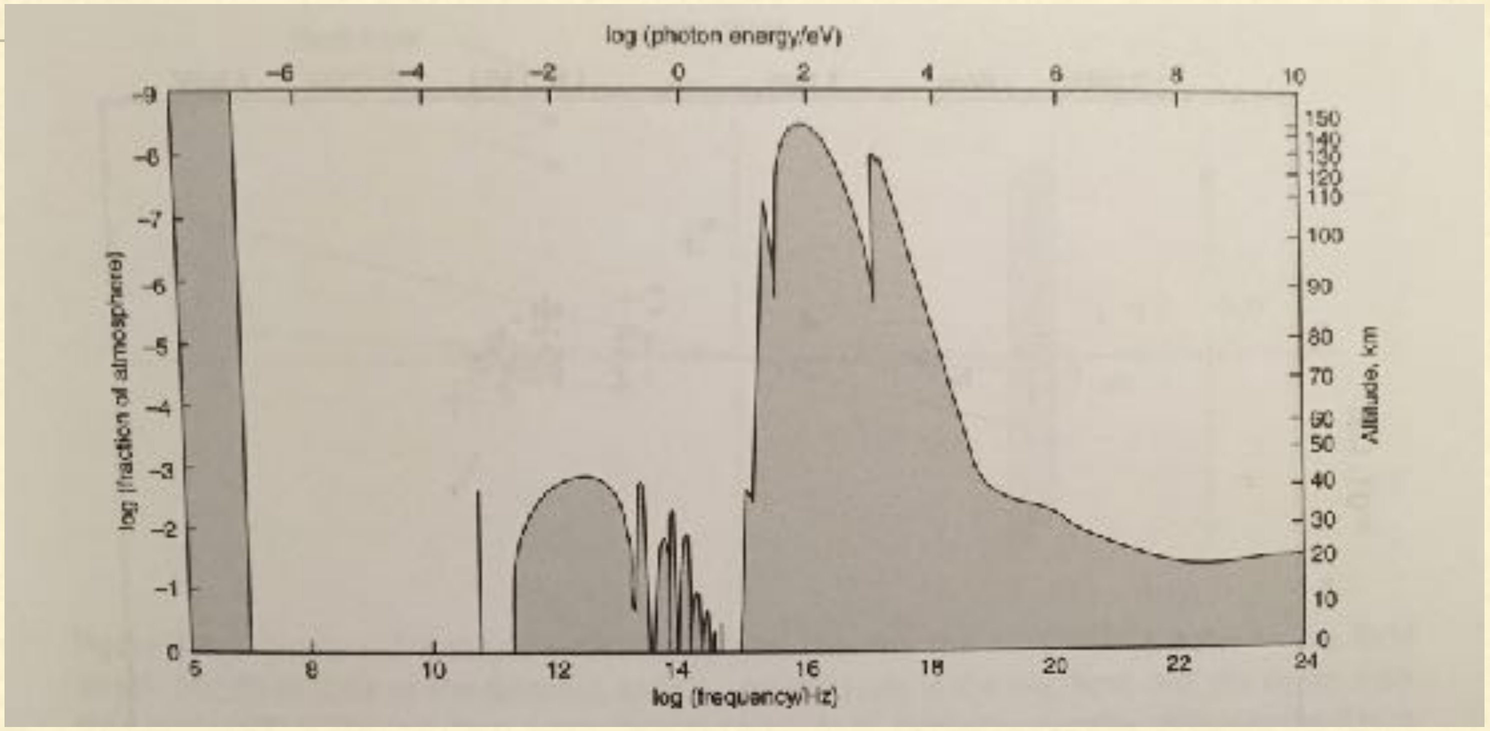
<u>But</u> hard to detect! $B > 3.5\,G$ to be detectable

ν_0 very low beyond the ionospheric cutoff

$B = 1\,G$, $\Rightarrow$ $\nu_0 = 2.8\,\text{MHz}$ not observable from ground.

pulsars, Sun,

planets w/ magn fields



Jupiter

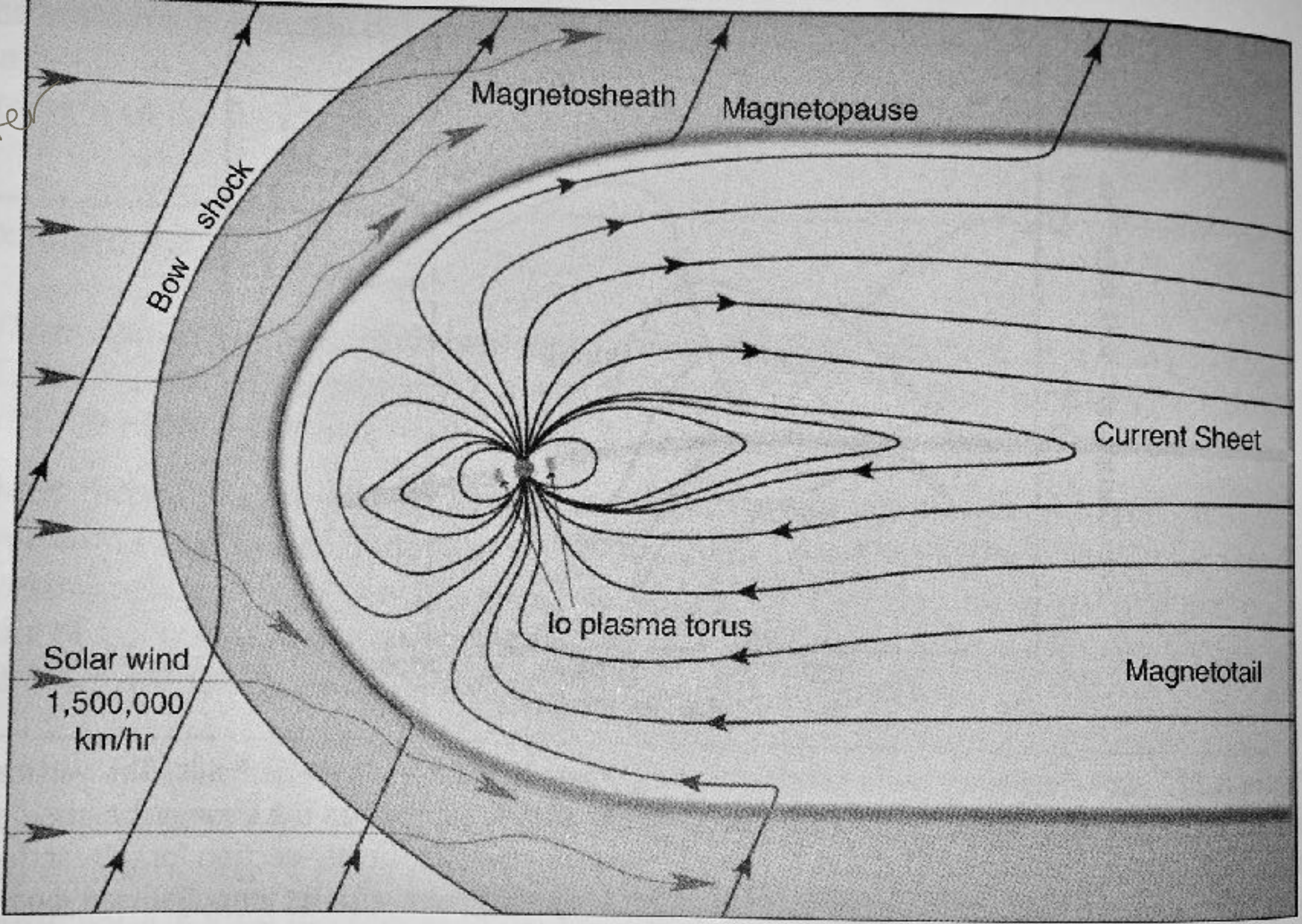


Figure 8.13. Diagram of Jupiter's *magnetosphere*. The planet, itself, is the small dot near the centre. The curves with arrows show the field lines which come closer together (stronger fields) near the poles of the planet. Note the Io plasma torus as well as the asymmetric shape of the field due to its interaction with the Solar wind. (Adapted from http://pluto.jhuapl.edu/science/jupiterScience/magnetosphere.html)

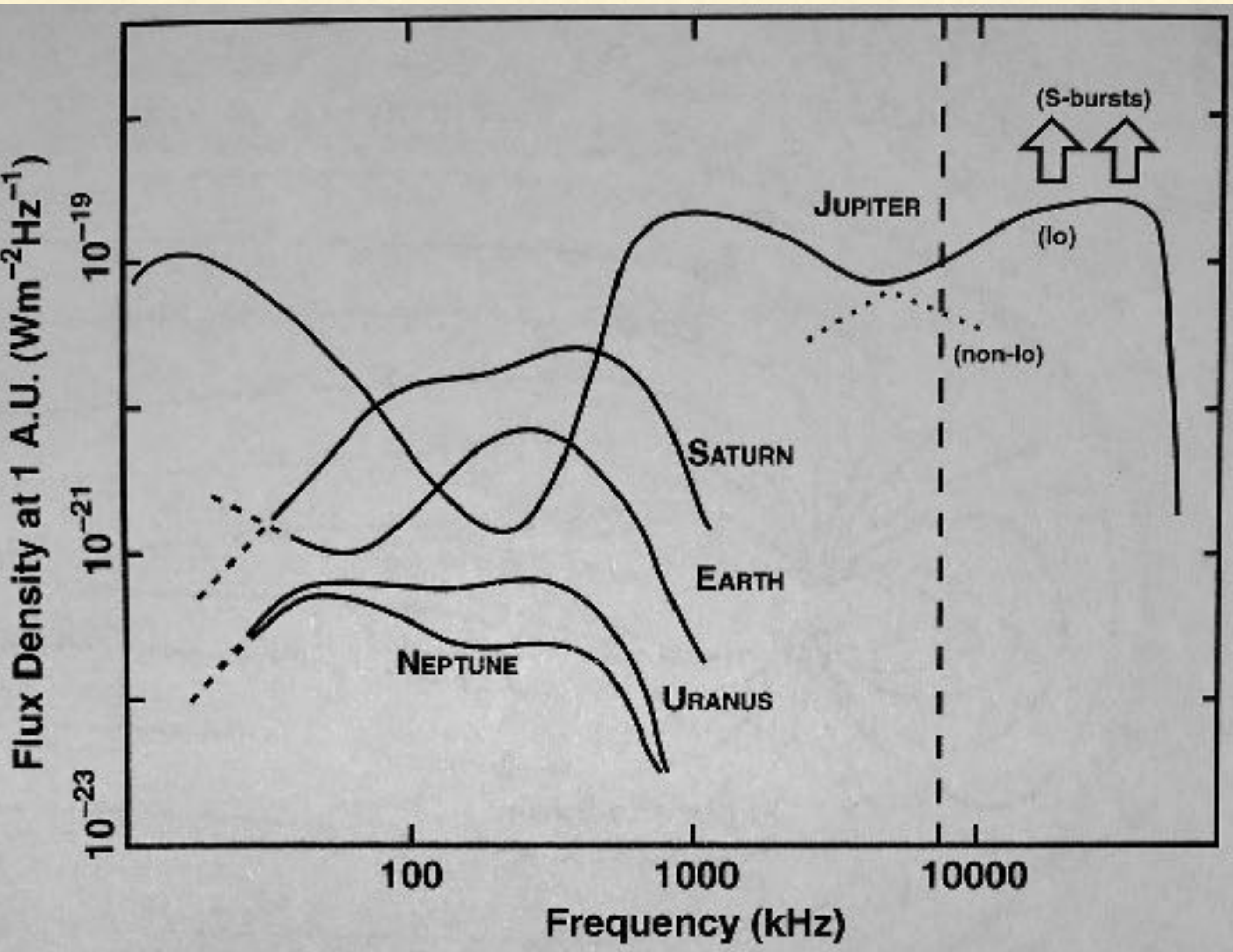


Figure 8.12. Low frequency radio spectra of the planets that have magnetic fields. The vertical dashed line marks approximately the Earth's ionospheric cutoff. Jupiter has the strongest emission, having a higher magnetic field (Table 8.1) and a larger magnetospheric cross-section for interaction with Solar wind particles. The interaction of Jupiter's magnetosphere with its inner Galilean moon, Io, is responsible for higher frequency emission from this planet as well as the 'S-bursts' which refer to short duration radio bursts. During radio bursts, peak flux densities can be $>10\times$ higher than shown here and, for Jupiter, $>100\times$ higher. The flux density scale assumes that the radio emission has been measured at a distance of 1 AU from each planet (Adapted from Ref. [187]).

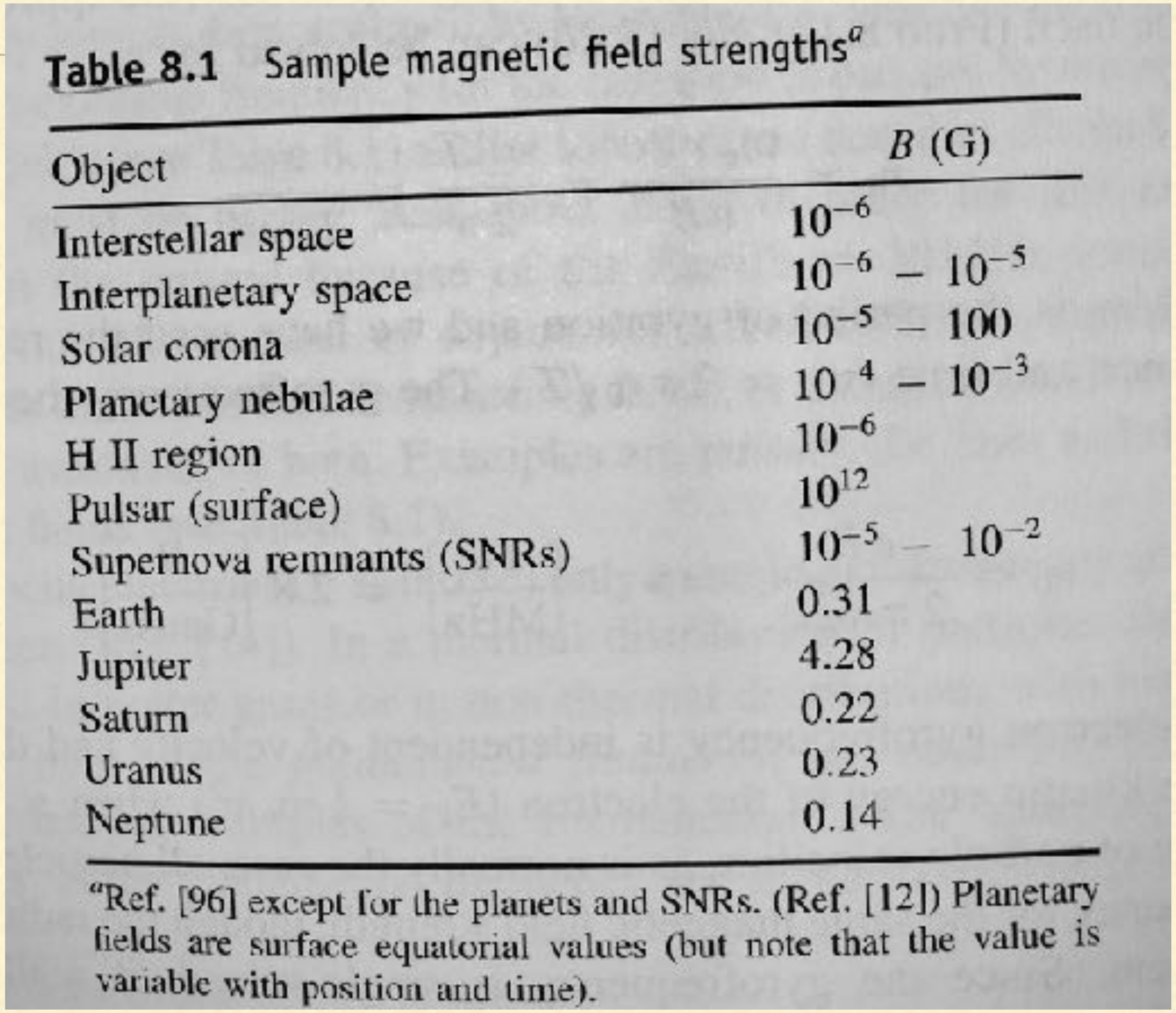
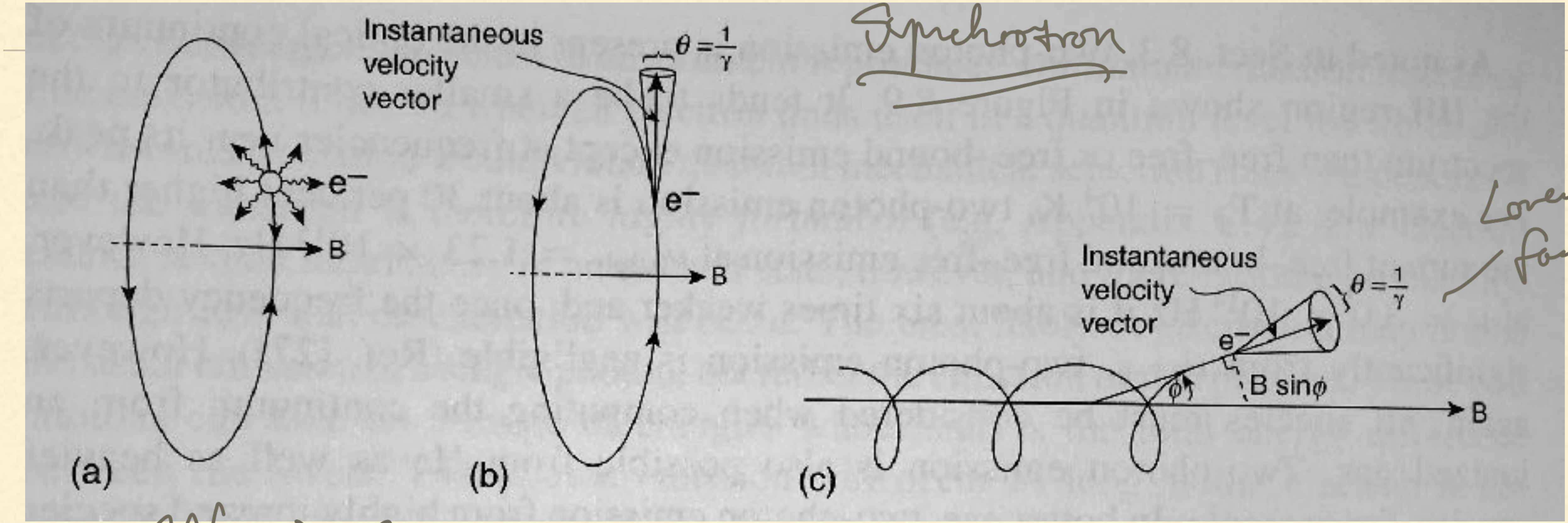


Table 8.1 Sample magnetic field strengths[a]

Object	B (G)
Interstellar space	10^{-6}
Interplanetary space	$10^{-6} - 10^{-5}$
Solar corona	$10^{-5} - 100$
Planetary nebulae	$10^{-4} - 10^{-3}$
H II region	10^{-6}
Pulsar (surface)	10^{12}
Supernova remnants (SNRs)	$10^{-5} - 10^{-2}$
Earth	0.31
Jupiter	4.28
Saturn	0.22
Uranus	0.23
Neptune	0.14

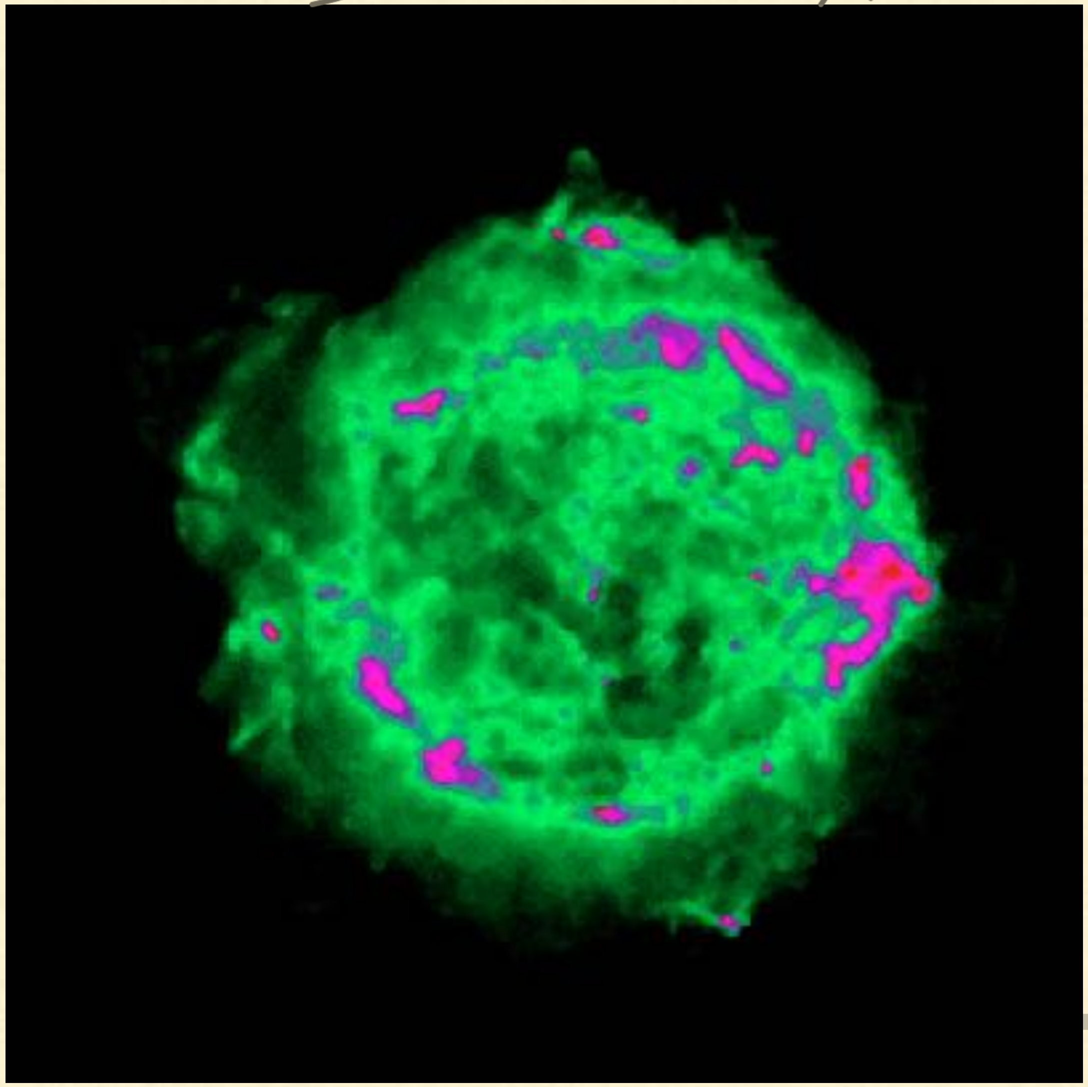
[a] Ref. [96] except for the planets and SNRs. (Ref. [12]) Planetary fields are surface equatorial values (but note that the value is variable with position and time).

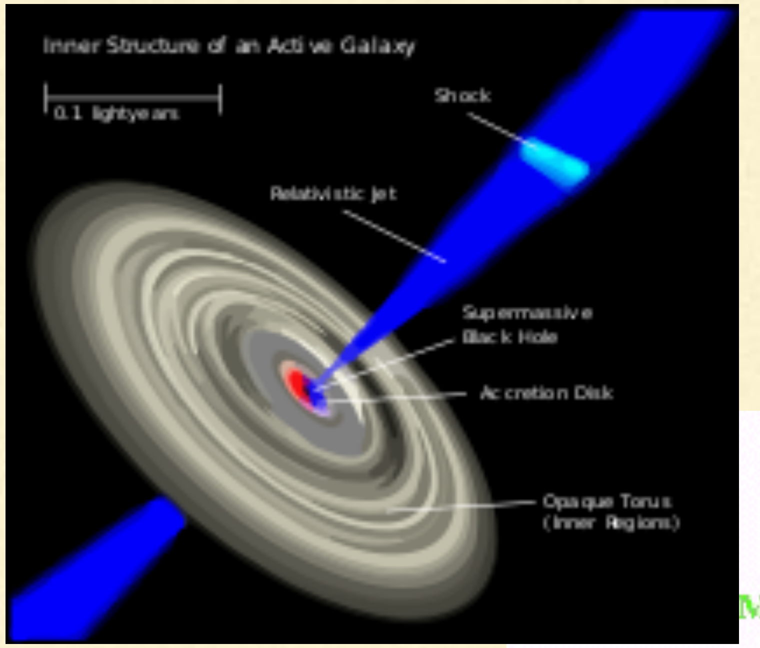
Synchrotron



Lorentz-factor

SNe & Cas A

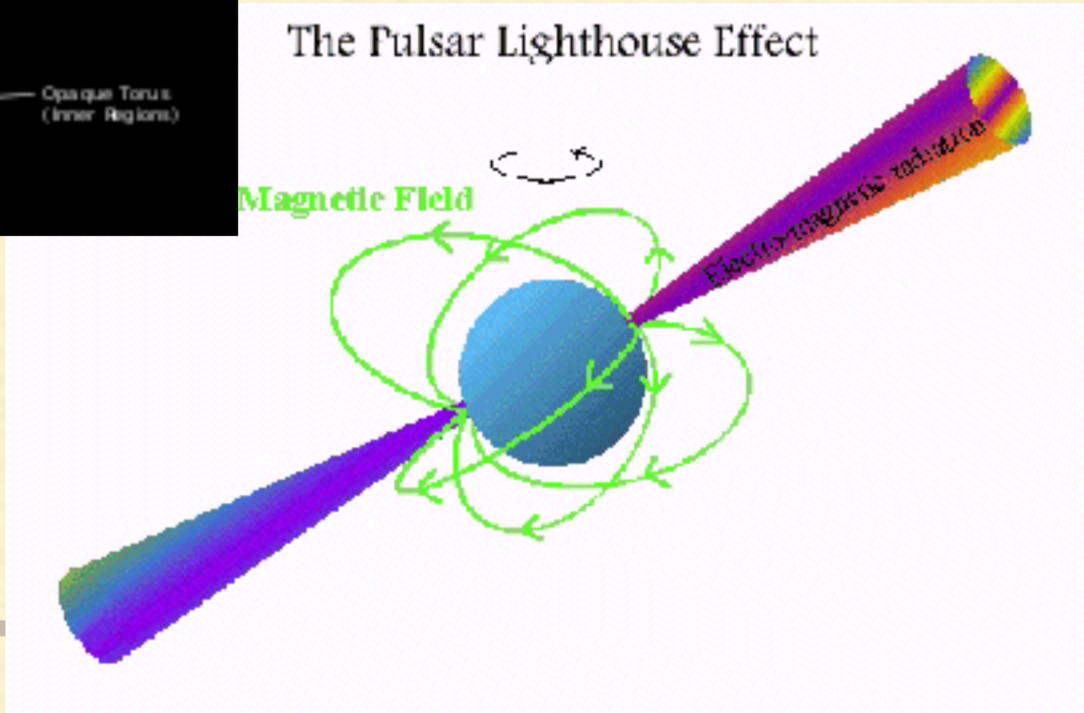


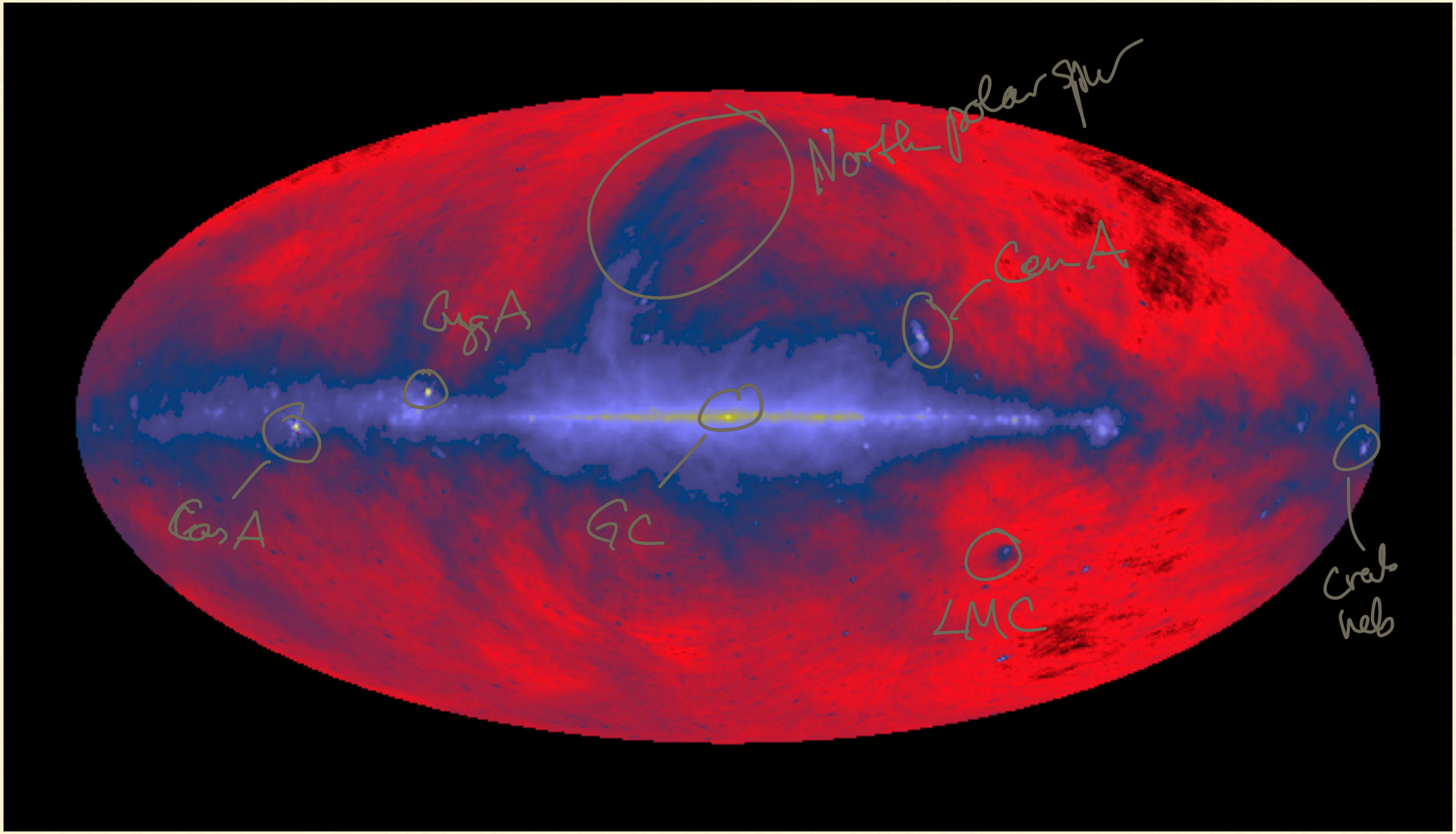


Cosmic rays = rel e^-

AGN

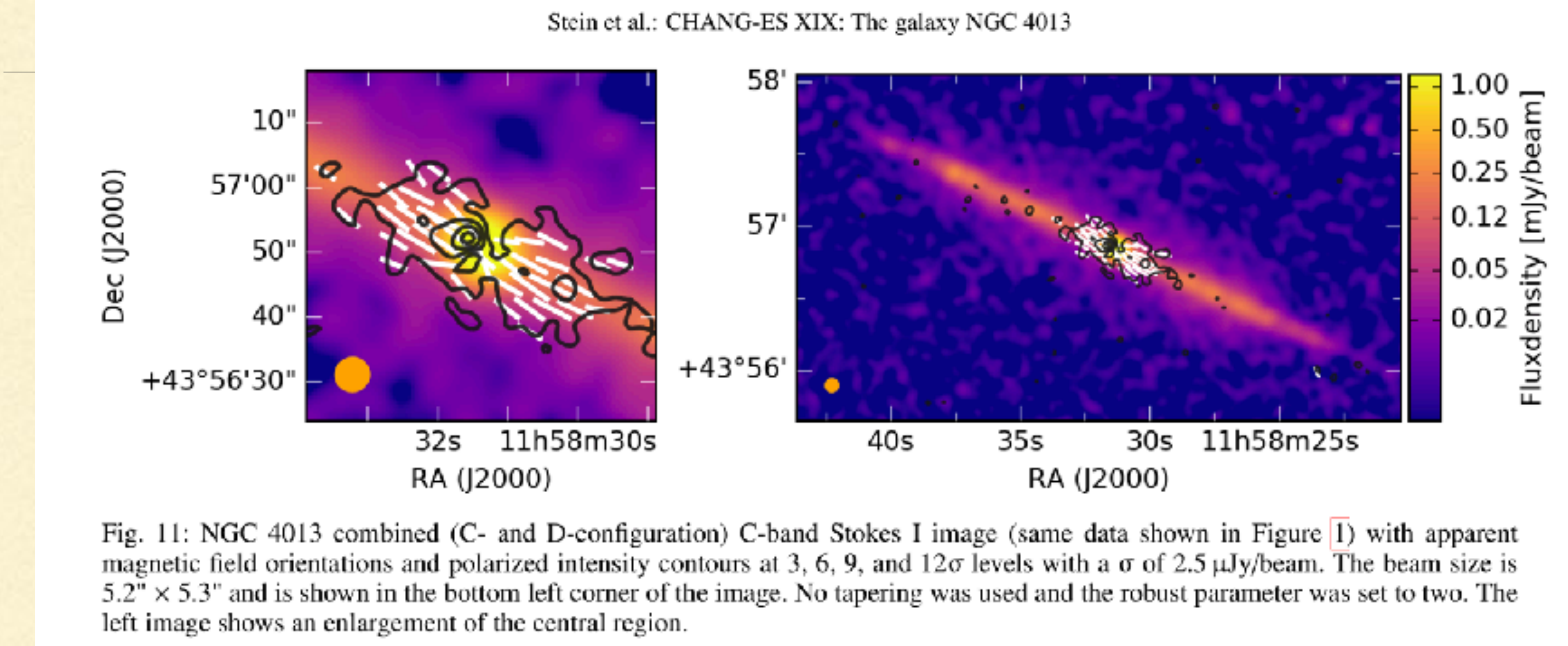
pulsars





408 MHz 1970's

Synchrotron radiation is intrinsically polarized.



Stein et al.: CHANG-ES XIX: The galaxy NGC 4013

Fig. 11: NGC 4013 combined (C- and D-configuration) C-band Stokes I image (same data shown in Figure 1) with apparent magnetic field orientations and polarized intensity contours at 3, 6, 9, and 12σ levels with a σ of 2.5 μJy/beam. The beam size is 5.2" × 5.3" and is shown in the bottom left corner of the image. No tapering was used and the robust parameter was set to two. The left image shows an enlargement of the central region.

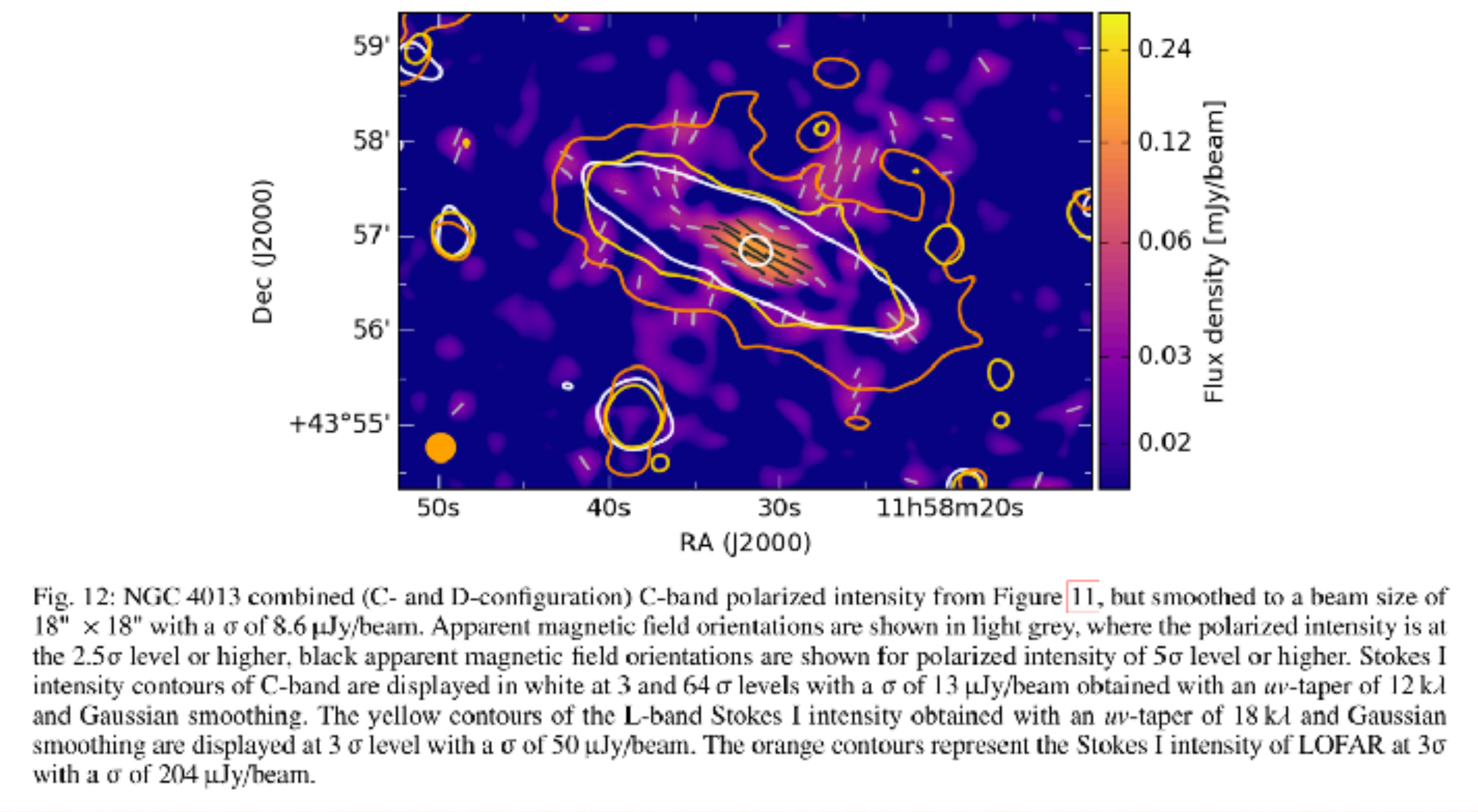


Fig. 12: NGC 4013 combined (C- and D-configuration) C-band polarized intensity from Figure 11, but smoothed to a beam size of 18" × 18" with a σ of 8.6 μJy/beam. Apparent magnetic field orientations are shown in light grey, where the polarized intensity is at the 2.5σ level or higher, black apparent magnetic field orientations are shown for polarized intensity of 5σ level or higher. Stokes I intensity contours of C-band are displayed in white at 3 and 64 σ levels with a σ of 13 μJy/beam obtained with an uv-taper of 12 kλ and Gaussian smoothing. The yellow contours of the L-band Stokes I intensity obtained with an uv-taper of 18 kλ and Gaussian smoothing are displayed at 3 σ level with a σ of 50 μJy/beam. The orange contours represent the Stokes I intensity of LOFAR at 3σ with a σ of 204 μJy/beam.

e^- relativisti — Lorentz will be involved in all

$$m = m_0 \gamma \qquad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$E = \gamma m_0 c^2$$

eq valid for cyclotron ($v \ll c \Rightarrow \gamma = 1$)

$$r = \gamma r_0$$

$$\Rightarrow r = \frac{m_e \overset{v \to c}{v_\perp} c}{eB} = \frac{\gamma m_e c^2}{eB} = \frac{E}{eB} = 2{,}2 \cdot 10^9 \frac{\overset{\text{erg}}{E}}{\underset{\text{G}}{B}} \quad \text{eq } 8.29$$

and gyrofrequency

$$\nu = \frac{\nu_0}{\gamma} = \left[\nu_0 = \frac{eB}{2\pi m_e c}\right] = \frac{eB}{2\pi \gamma m_e c} = 2{,}3 \frac{B}{E}$$

↑ energy of E

These eq now $\propto E$ (unlike cyclotron)

γ can be very high $\sim 10^4$ or so ⇒

rel $r \gg$ Cyclotron r_0

so .... ν much too low to measure?

Ex 8.4 get r, ν & T of e^- in ISM, $B = 3\,\mu G$

gyroradius gyro ν $\frac{1}{\nu}$ $\gamma = 10^4$

$$\Rightarrow E = \gamma m_e c^2 = 8{,}18 \cdot 10^{-3}\ \text{erg}$$

$$r = 2{,}2 \cdot 10^9 \frac{E}{B} = 6 \cdot 10^{12}\ \text{cm} = 86 \cdot R_\odot$$

$$\nu = 2{,}3 \frac{B}{E} = 8.4 \cdot 10^{-4}\ \text{Hz} \qquad T = 20\ \text{min}$$

$\nu_{obs} \neq \nu_{gyro}$

$\Rightarrow$ Lorentz transformations

opening angle of cone $\propto E$, higher $E, \nu \Rightarrow$ narrower cone

/ avg radius of cone $\theta = \frac{1}{\gamma}$

direction of cone follows direction of $\mathbf{v}$

$\Rightarrow$ cone sweeps by the los

high vel $\Rightarrow$ small opening angle $\rightarrow$ pulses of emission short duration τ

shortest τ (pulse duration) $\Rightarrow$ max ν as seen by you

$\Rightarrow$ critical frequency (above ν_{crit}, emission negligible)

$$\nu_{crit} = \frac{3}{2}\gamma^2 \nu_0 \sin\varphi = \frac{3e}{4\pi m_e c}\gamma^2 B_\perp \Rightarrow$$

$$\left[\frac{\nu_{crit}}{MHz}\right] = 4{,}2\,\gamma^2 \left[\frac{B_\perp}{Gauss}\right] \qquad \text{eq } 8.32$$

∴ max γ is high $\gg$ relativistic gyrofrequency

$\nu_{max} = 0{,}29\,\nu_{crit}$ ← this is where most energy emitted, a bit lower than ν_{crit}